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ATMOSPHERIC TURBULENCE ENVIRONMENT WITH SPECIAL REFERENCE TO CONTINUOUS TURBULENCE

by

HARRY PRESS

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This Report was presented at the Fifth Meeting of the Structures and Materials Panel, held from 29th April to 3rd May, 1957, in Copenhagen, Denmark

SUMMARY

The flight-test techniques for measuring continuous atmospheric turbulence are briefly outlined and available measurements of the power spectrum of atmospheric turbulence reviewed. The measured results are examined in order to determine how well they approximate the idealized models of a stationary Gaussian random process and of isotropic turbulence. Finally, some recent results on the variation with altitude of the probability distribution of the root-mean-square gust velocity are presented and the application of these results to operational response history calculations indicated.

SOMMAIRE

Exposé sommaire des techniques d'essais en vol permettant de mesurer la turbulence atmosphérique continue et revue des mesures existantes relatives au spectre énergétique de la turbulence atmosphérique. Etude des résultats obtenus en vue de déterminer jusqu'à quel point ils s'approchent des cas idéalisés d'un phénomène fortuit Gaussien stationnaire et de la turbulence isotropique. En conclusion, présentation de quelques résultats récemment acquis concernant la variation, avec l'altitude, de la distribution des probabilités de la vitesse efficace des rafales et application de ces résultats aux calculs des réponses de l'avion dans des conditions d'utilisation.

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NOTATION

a_n	acceleration
$\bar{A}$	see Equation (14) in main text
a_x	airplane longitudinal acceleration
$\bar{c}$	average chord, ft
$I(K, s)$	airplane gust response function, Equation (15) in main text
K	$\frac{4W}{g\pi\rho S\bar{c}}$
$h(t)$	unit impulse response
$H(\omega)$	frequency response function
l	distance from vane to airplane center of gravity, ft
L	scale of turbulence, ft
m	slope of lift curve per radian
$\overline{N(a_n)}$	average number of maximums per second exceeding given value of a_n in operations
N_0	characteristic frequency, Equation (11a) in main text
$N(\ )$	average number of maximums per second exceeding given value of specified argument for Gaussian disturbances
O_1, O_2	output responses
$p(\sigma)$	probability distribution of σ
$\hat{p}(\sigma_w)$	probability distribution of σ_w
S	wing area, sq ft
t	time, sec
u	longitudinal gust velocity, ft/sec
v	lateral or side gust velocity, ft/sec
V	airplane forward speed, ft/sec
w	vertical gust velocity, ft/sec

W	airplane weight, lb
w_a	airplane normal or vertical velocity, ft/sec
α	angle of attack, radians
γ	flight-path angle, w_a/V
θ	pitch attitude, radians
θ	pitch velocity, radians/sec
λ	gust wave length, ft
ρ	air density, slugs/cu ft
σ	root-mean-square deviation
$\Phi(\omega)$	power-spectral-density function
Ω	reduced frequency (ω/V), radians/ft
ω	frequency, radians/sec

Subscripts

i	input
o	output

ATMOSPHERIC TURBULENCE ENVIRONMENT WITH SPECIAL REFERENCE TO CONTINUOUS TURBULENCE

Harry Press*

1. INTRODUCTION

Aeronautical studies of atmospheric turbulence have, in the past, been largely based on simplified concepts involving discrete gusts of idealized shapes such as sharp-edged gusts, triangular gusts, and ramp-platform shaped gust profiles. During the last few years, a number of developments have contributed to making a more detailed and realistic approach to the rough-air problem more urgent. These include, on the one hand, the increased importance of airplane response to gusts in stability and guidance problems. On the other hand, the need for the rapid exploitation of airplane performance capabilities has served to place greater emphasis on a more efficient structural analysis. As a consequence, the gust response, which was formerly of serious concern only for large transport-type airplanes and bombers, has also become of concern for a much wider range of airplane types as well as for missiles. Fortunately, during this period, new mathematical and experimental tools have been developed which served to make a more detailed study of atmospheric turbulence feasible. Noteworthy developments occurred in the theory of random processes, in the statistical theories of turbulence, and in methods for the measurements of turbulence in flight, and these developments have served to provide the basic tools for a deeper approach to gust problems.

As a consequence of these new developments, the last 5 years have seen a rapid development in the study of continuous atmospheric turbulence and airplane behavior in rough air. The detailed characteristics of continuous atmospheric turbulence have been measured in a number of studies. These measurements have served to provide a new insight into airplane behavior under continuous rough-air conditions. References 1 - 25 list some of the more important contributions in these areas.

Although the number of detailed measurements of continuous atmospheric turbulence is still quite limited, sufficient data appear to be available to warrant a review of these measurements and at least a preliminary assessment of the general characteristics of atmospheric turbulence. In the present paper, the general methods used in the measurement of continuous atmospheric turbulence in flight will be reviewed briefly and an effort made to summarize some of the more significant features of the information available on the characteristics of atmospheric turbulence. In order to provide a compact description of turbulence measurements and to provide a guide to the selection of simple models for analytic studies, some of the basic concepts and results of random process theory and the statistical theories of turbulence will be described in the appendix to this paper. The important concepts of a stationary, a homogeneous, and a Gaussian random process will be defined, and the characteristics of isotropic turbulence will be described. These idealized models are used in the assessment of the experimental measurements of turbulence in the main body of this paper.

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In addition to the foregoing material which is confined to the local characteristics of atmospheric turbulence, consideration will also be given to some of the broader climatological variations in turbulence. In particular, some recently obtained estimates of the probability distribution of the root-mean-square gust velocity and its variation with altitude are presented and discussed. The application of these results to the problem of estimating gust and gust load histories in airplane operations is also considered briefly.

2. METHODS OF MEASURING ATMOSPHERIC TURBULENCE IN FLIGHT

The need for the consideration of the continuous character of turbulence has, in recent years, led to the development of new and improved methods of measuring turbulence in flight. During the last few years, two significant contributions were made in this area. These are: first, the general approach pioneered by Clementson, at Massachusetts Institute of Technology[1] for the determination of power spectra of atmospheric turbulence by means of power spectral techniques; and secondly, the subsequent development, at both the Cornell Aeronautical Laboratory[21,24] and the N.A.C.A.[11,23] of direct turbulence measurements by means of flow direction vanes. These techniques have been used to provide basic data on the detailed characteristics of turbulence. A few remarks on these methods and their limitations thus appear appropriate in order to appreciate the limitations of the available information.

2.1 Spectral Technique

In Reference 1, Clementson first applied the concepts of random process theory (see Appendix) to the problem of airplane behavior in rough air. In this approach, the power spectra of the random disturbance and of the airplane response play a central role. Clementson showed how output response measurements of a linear airplane system could be used to determine the power spectrum of atmospheric turbulence. The approach used is based on the application of the relation between the power spectrum of a disturbance and the power spectrum of the response to the disturbance for a linear system. The relation between the power spectra is given by

$$\Phi_O(\omega) = \Phi_i(\omega)\,|H(\omega)|^2 \qquad (1)$$

where

ω = circular frequency, radians/sec

$\Phi_O(\omega)$ = the power spectrum of the response or output O(t)

$\Phi_i(\omega)$ = the power spectrum of the input or gust disturbance I(t)

$H(\omega)$ = the frequency response function for the system and in the present case defines the response of the system to unit sinusoidal gusts at the various frequencies.

The application of this relation to the measurement of gust spectra is schematically illustrated in Figure 1. The reduced frequency argument Ω ($=\omega/V$) which is equal to

$2\pi/\lambda$ where λ is the gust wavelength, is used for the abscissa inasmuch as the space variations of the turbulence are of principal concern in aeronautical applications. The upper curve represents the power spectrum of an airplane response such as airplane normal acceleration. The second curve represents the airplane response characteristics, and the sketch shown in typical of the normal acceleration response at the center of gravity of some current airplane types. The frequency response function normally contains several peaks which reflect the various airplane response modes. For the case shown, the first peak is a reflection of the airplane short-period mode which normally peaks at around 1/2 cycle/sec, or a corresponding gust wavelength of the order of 1,000 ft. The second peak shown on the sketch is a reflection of the first mode in sym-·rical bending which frequently is present at 2 to 5 cycles/sec and corresponds to gust wavelengths of the order of 100 ft. Thus, wavelengths of from less than 100 ft to several thousand feet are of principal concern in many aeronautical applications. Special problems, such as missile control problems, frequently involve consideration of even longer gust wavelengths. The last curve is obtained by dividing the response spectrum by the amplitude squared of the airplane frequency response function and yields the power spectrum of vertical gust velocity.

It is clear from Equation (1) that the power spectrum of the input may be determined from measurements of the power spectrum of an airplane response alone, if the frequency response function for the airplane is known. This is essentially the procedure first used by Clementson and subsequently by others. In these applications, the general practice has been to determine the frequency response function by analytical means. Equation (1) may, of course, also be used to calculate responses for known inputs, and this application is the central one in aeronautical applications and has found wide usage.

Later investigators, notably Lappi, at the Cornell Aeronautical Laboratory, and Summers, at Massachusetts Institute of Technology[9], have extended this approach to the case of simultaneously determining several components of the turbulence such as the vertical and head-on or longitudinal components. This extension requires the simultaneous measurement of several output responses, each one being significantly affected by the several turbulence components.

For two disturbances or gust components, $I_1(t)$ and $I_2(t)$, the relations for the output responses are given by

$$\left.\begin{aligned} O_1(t) &= \int_{-\infty}^{\infty} h_{11}(t_1)I_1(t - t_1)dt_1 + \int_{-\infty}^{\infty} h_{12}(t_1)I_2(t - t_1)dt_1 \\ O_2(t) &= \int_{-\infty}^{\infty} h_{21}(t_1)I_1(t - t_1)dt_1 + \int_{-\infty}^{\infty} h_{22}(t_1)I_2(t - t_1)dt_1 \end{aligned}\right\} \quad (2)$$

where $h_{ij}(t)$ is the unit impulse response in O_i to the disturbance I_j. The relations between the power spectra and cross spectra (see Appendix) of the disturbances and the responses for each frequency may best be expressed in matrix form as

$$\begin{Bmatrix} \hat{\Phi}_{11} \\ \hat{\Phi}_{21} \\ \hat{\Phi}_{12} \\ \hat{\Phi}_{22} \end{Bmatrix} = \begin{bmatrix} |H_{11}|^2 & H_{11}H_{12}^* & H_{11}^*H_{12} & |H_{12}|^2 \\ H_{11}H_{21}^* & H_{11}H_{22}^* & H_{21}^*H_{12} & H_{22}^*H_{12} \\ H_{11}^*H_{21} & H_{21}H_{12}^* & H_{22}H_{11}^* & H_{22}H_{12}^* \\ |H_{21}|^2 & H_{22}^*H_{21} & H_{22}H_{21}^* & |H_{22}|^2 \end{bmatrix} \begin{Bmatrix} \Phi_{11} \\ \Phi_{21} \\ \Phi_{12} \\ \Phi_{22} \end{Bmatrix} \quad (3)$$

where

Φ_{ij} = the power spectra of the input disturbances if $i = j$ or the cross spectra between the input disturbances if $i \neq j$

$\hat{\Phi}_{ij}$ = the power spectra of the outputs if $i = j$ or the cross spectra between the outputs if $i \neq j$

$H_{ij}(\omega)$ = frequency response function defining the response in O_i to unit sinusoidal disturbances in I_j at the various frequencies and is given by

$$H_{ij}(\omega) = \int_0^\infty h_{ij}(t)e^{-i\omega t}dt.$$

The asterisk designates the complex conjugate. If the H_{ij}'s are known, measured values of the $\hat{\Phi}_{ij}$'s may be used in Equation (3) to determine the required disturbance spectra and cross spectra Φ_{ij}. The extension to three or more components is straightforward.

The principal difficulty with this approach for the determination of atmospheric turbulence spectra is the requirement for reliable estimates of the airplane frequency response functions over the wide range of frequencies which are normally of concern. Calculated frequency response functions can generally be obtained, but their reliability is not yet considered adequate to warrant the general use of this approach for the present purpose. Experimental methods of determining the frequency response functions of an airplane for the gust condition appear to be promising although they are still in an early state of development. An additional complication is introduced by the unavoidable presence in flight tests of pilot control motions. As a consequence, more direct methods of measuring turbulence that are independent of the airplane frequency response functions are desirable. The flow direction vane technique described in the next section appears to meet this need.

2.2 Flow Direction Vane Technique

The use of flow direction vane measurements for the study of turbulence was developed independently at Cornell Aeronautical Laboratory[21, 24] and by Chilton and Crane at the N.A.C.A.[11, 23]. Figure 2 illustrates schematically the relations between the flow direction and the gust velocity for an airplane.

The airplane angle of attack α_a is given by

$$\alpha_a = \theta + \gamma \tag{4}$$

where

θ = the pitch attitude, radians

γ = w_a/V

w_a = the airplane normal velocity, ft/sec

V = the airplane forward speed, ft/sec.

For small disturbances, the angle sensed by the vane α_v, neglecting the upwash, is then given by

$$\alpha_v = \alpha_a + \alpha_g - \frac{l\dot{\theta}}{V} \tag{5}$$

where

$\alpha_g = \frac{w_g}{V}$

w_g = the normal (or vertical) gust velocity, ft/sec

l = the distance from the vane to the airplane center of gravity.

From Equations 4 and 5, the vertical gust velocity is then given by

$$w_g = V(\alpha_v - \theta) - w_a + l\dot{\theta} \tag{6}$$

Equation 6 indicates that the time history of gust velocity sensed by the airplane can be determined from the simple addition of measurements of airplane vane angle, pitch attitude, airplane vertical velocity, and pitch velocity. The use of the equivalent expression for the reduction of sideslip angle measurements to obtain the side gust component does not appear to offer any additional serious difficulties.

In principle, the application of Equation (6) to gust measurements is straightforward. In practice, a number of difficulties are encountered which tend to limit the accuracy and the range of frequencies which can be covered. These difficulties stem essentially from limitations in measurement techniques. As might be expected, the instrument accuracy requirements for such applications are quite severe. For an airplane flying at 300 ft/sec, a 1-ft/sec vertical gust gives rise to an angle-of-attack change of 1/300 of a radian, or roughly, 0.2°. It is thus clear that in order to achieve an accuracy of 1 ft/sec, it is necessary to measure the required angles to at least an accuracy of 0.1° or 0.05°. This high degree of sensitivity is difficult to achieve but can be approximated in practice when some precautions are taken. In addition, good frequency response characteristics to relatively high frequencies are required in order to avoid distortions due to magnifications and phase shifts.

Flow direction vanes with good frequency response characteristics do not appear difficult to obtain. Standard metal vanes frequently have response characteristics that are essentially flat to 10 cycles/sec. Lightweight balsa-wood vanes with good response to much higher frequencies are not difficult to obtain. Angle-of-attack pressure sensors have also been considered. Perhaps the most troublesome of the measurements is the measurement of pitch attitude. Although good frequency response characteristics can be obtained, conventional pitch attitude gyro recorders are subject to low frequency drift. This difficulty can largely be avoided by obtaining pitch attitude from the integration of pitch velocity measurements. Some investigators have had recourse to the use of a sun-camera technique for studies covering very low frequencies[23]. Airplane vertical velocity is generally difficult to measure directly but the integration of normal acceleration measurements appears to provide a satisfactory measure of vertical velocity when the initial conditions are adjusted through the use of a sensitive pressure altimeter or statoscope.

An additional practical difficulty is worth noting. The necessity for getting the vane measurements out of the airplane flow field, in general, requires the use of a vane mounted on a boom. Booms mounted at the airplane center line have appeared best for rigidity reasons and have the additional advantage of avoiding the complications of the effects of rolling and yawing motions. Boom lengths of the order of one chord length have been used and appear to be satisfactory. However, in practice, the boom stiffness is generally not as good as is desirable. Boom vibrational frequencies thus become a problem, but it is generally possible to keep the boom frequencies sufficiently high (5-15 cycles/sec) to be out of range of the more significant gust frequencies which are generally at the lower frequencies.

The sensitivities used in recording the various quantities can considerably influence the reliability of the spectrum measurements. Because of the general character of the gust spectrum and the spectra of the various other airplane measurements, the measurements will, in general, contain very little power at the higher frequencies. Thus, the results obtained at the higher frequencies particularly are susceptible to large errors. As a consequence, studies aimed at covering a wide range of frequencies may require different sensitivities and particularly higher sensitivities for the higher frequency ranges. The fact that w_a and θ are determined from the integration of acceleration and pitch velocity measurements makes the low frequency accuracy of w_a and θ critical since small errors in say $\dot{\theta}$ (orΔn)at low frequencies yield larger errors in derived values of θ (or w_a).

The numerical determination of the power spectra from the time history data is, in general, a task of some delicacy. Care is required in the choice of reading intervals, and length of sample record in order to obtain reliable results. The actual numerical operations in the determination of the auto-correlation function and power spectrum, as defined in Equations (3) and (4) of the Appendix, must also be carefully planned to avoid distortions arising from the limited data and from the numerical operations themselves. These problems are particularly important in the gust spectra determinations and are discussed in some detail in Reference 20.

The full expression given by Equation (6) is not required over the full frequency range of interest which covers the wavelength range of 1 ft to perhaps 50,000 ft. In portions of this wide range of wavelengths or frequencies, some of the terms become negligible. At high frequencies, say above 2 or 3 cycles/sec for many current airplane types, the airplane pitch and vertical motion response are generally small relative to the gust angle and can be neglected. For this case, Equation (6) reduces to

$$w_g = V\alpha_v \tag{7}$$

and the vane measurement alone provides a direct measure of the turbulence. At very low frequencies, generally less than about 0.25 cycles/sec, the airplane's inherent stability acts to keep the airplane at a fixed angle of attack, and the vane measurements are then of no use. The gust velocity for these low frequencies is then given by

$$w_g = -V\theta - w_a \tag{8}$$

and, in general, depends upon small differences between the vertical motion and the pitching motion. The accuracy requirements on these two measurements consequently become more critical. The reliability of conventional pitch-rate gyros is somewhat questionable at these lower frequencies, and pitch attitude measurements, using a sun-camera technique, have for this reason been used in this frequency region[23].

In order to provide an indication of the magnitude of the corrections introduced by the various airplane motions in the airplane short-period frequency region, some recent results obtained in connection with a study of the behavior of a large swept-wing airplane in rough air are summarized in Figure 3. Figure 3 shows the basic spectrum of $V\alpha_v$ (the vane indicated gust velocity). The modified gust velocity spectra obtained when the corrections for vertical motions ($V\alpha_v - w_a$) and when the corrections for vertical motions and pitching motions ($V\alpha_v - V\theta - w_a + l\dot{\theta}$) are included, are also shown.

Figure 3 indicates that the corrections for the airplane motions tend to increase the gust spectrum at very low frequencies ($\Omega < 0.003$) indicating that the airplane pitching and vertical motions act to reduce the airplane angle of attack due to the gust. At the higher frequencies ($0.003 < \Omega < 0.010$) the airplane motions increase the airplane angle of attack above the gust angle-of-attack change. This is largely a consequence of the pitching motions in the short-period frequency region. The magnitude of the corrections is quite large in this region reducing the gust spectrum to about one-third that indicated by the vane angle alone. At still higher frequencies, the difference between the vane indicated gust velocity and the full expression is seen to be small, although, in the present case, the situation is complicated by the presence of vibration effects associated with the airplane fundamental vibration mode which has a frequency corresponding to about $\Omega = 0.012$ radians per ft. It is thus clear from the foregoing that the correction for pitching and vertical motions is quite large in the airplane short-period region and is required for reliable gust spectra.

In spite of the many difficulties encountered in practice, the flow direction technique for turbulence measurement appears to provide the most reliable method so far devised for the measurement of atmospheric turbulence in flight. The experimental results to be described subsequently will be largely restricted to such measurements.

2.3 Other Methods

Two other methods of measuring turbulence deserve mention and will be discussed briefly.

Inasmuch as pressure measurements in flight are a well-developed art, the high-frequency fluctuations of a sensitive and high-frequency airspeed system provide a useful method of measuring the longitudinal (head-on) component of turbulence. The longitudinal gust velocity u(t) would appear to be given simply by

$$u(t) = \Delta V(t) - \int_0^t a_x(t_1)dt_1 \qquad (9)$$

where

ΔV = the fluctuation of the measured airspeed, ft/sec

a_x = the longitudinal airplane acceleration, ft/sec.

Because of the high airplane inertia, the acceleration a_x is frequently negligible at the higher frequencies and the fluctuations of the airspeed may sometimes be taken as a direct measure of the turbulence. For many airplanes, this appears to be reliable for gust frequencies down to at least 1 cycle/sec. The longitudinal gust velocities have not, however, yet received much detailed consideration inasmuch as their effects on the airplane are generally small.

A large body of atmospheric turbulence measurements has been collected in recent years by meteorologists using wind measuring equipment mounted on towers. These measurements are, of course, restricted to the lower several hundred feet of the atmosphere. Also, the direct applicability of these measurements to aeronautical questions appears, at this time, to be open to some question, and for these reasons as well as for space limitations, it was decided to make no direct use of this material in the present study. Some qualitative references to these results will, however, be made.

3. CHARACTERISTICS OF ATMOSPHERIC TURBULENCE

3.1 General Remarks

In this section of the paper, some of the significant features of the available information on atmospheric turbulence are reviewed. No effort will be made to be exhaustive in covering the available data, but rather the intent will be to present illustrative results on the various questions of interest. The discussion will be divided into two parts, one concerned with what might be called the local detailed characteristics of atmospheric turbulence, the other concerned with the broader climatological variations of turbulence intensity.

In considering the detailed statistical characteristics of atmospheric turbulence, it will be helpful to determine how well the measurements can be approximated by simple theoretical models that have been developed both in random process theory and in general theories of turbulence. Such evaluations are important both for guiding analytic studies of airplane behavior which can be simplified by the use of idealized models and also for planning turbulence measurement surveys. The specific turbulence properties that are pertinent and are of present interest are:

(a) The power spectra of the component velocity fluctuations

(b) Homogeneity of the turbulence

(c) Stationarity of the turbulence

(d) Gaussianness of the velocity fluctuations

(e) Isotropy.

These properties are defined analytically in the appendix. It may, however, be helpful to consider briefly the significance and implications of these properties.

The power spectrum of a random process or a random disturbance in general provides a detailed description of the statistical characteristics of the disturbance, and, in particular, describes the frequency content of the disturbance. For the case of velocity fluctuations in turbulence with which we are concerned, the power spectrum describes how the kinetic energy is distributed with frequency or gust wavelengths. Inasmuch as airplanes are sensitive to different degress to turbulence of various wavelengths, this representation is particularly useful in aeronautical response studies.

The properties of homogeneity and stationarity imply a persistence and invariance in the statistical characteristics of the turbulence with space and time displacements, respectively. Thus, for a region of homogeneous turbulence, it would be expected that airplane measurements would yield the same statistical characteristics regardless of the starting point of the measurements. For stationary turbulence, the measurements would be invariant for different times of the surveys. The special case of problems involving stationary and homogeneous disturbances is particularly amenable to analysis and has received considerable attention.

The concept of a Gaussian disturbance is of particular importance in applications since it specifies the precise characteristics of the probability distributions associated with the disturbance. This specification tremendously simplifies the derivations in application of specific quantitative results of interest such as the number of peaks per second of various intensities (see paper by S.O. Rice in Reference 26).

Finally, the concept of isotropy is of interest since it may be viewed as the simplest type of turbulence. Isotropic turbulence has the property of invariance of the statistical characteristics of the turbulence with rotations of the reference axes. For aeronautical applications, isotropy implies that the statistical characteristics of the turbulence measured are independent of the airplane flight direction. A further important consequence of isotropy that is of extreme importance in turbulence considerations is the fact that isotropy specifies particular relations between the statistical characteristics of the three turbulence directional components. These relations are discussed in detail in Reference 27. A few of the most important ones are indicated in the Appendix.

3.2 Local Turbulence Characteristics

3.2.1 Time History Characteristics

In order to provide a general indication of the time history characteristics of atmospheric turbulence as measured by an airplane in flight, Figure 4 presents short concurrent time histories of the vertical gust velocity and the side or lateral gust velocity. For comparison, the associated airplane normal acceleration, which is sometimes used as a measure of the turbulence, is also shown in the figure. The first point of interest is the irregular and erratic character of the turbulence fluctuations, resembling a random type of oscillation. This characteristic, which is quite general, is of course the basic reason for the necessity for the applica-

tion of statistical techniques. Although there is some tendency for the normal acceleration to follow at least the more rapid vertical gust velocity changes, the two records are very different. Perhaps the major difference noticeable to the eye is the absence of the long waves in the acceleration record. This difference results from the airplane's lack of sensitivity to these longer gust wavelengths. Another point of interest is the lack of correlation between the vertical and side gust velocities.

3.2.2 *Power Spectrum of Turbulence*

A number of flight measurements of the spectrum of atmospheric turbulence have now been obtained at Cornell Aeronautical Laboratory, Massachusetts Institute of Technology, and the N.A.C.A. Additional studies are currently under way. These measurements reveal a rather surprising degree of uniformity in the general shape of the spectrum. For this reason, no effort will be made to present the many measurements available, but rather one set of measurements for vertical gust velocity is shown in Figure 5. This measured spectrum was obtained in Reference 23 and is essentially typical of most of the measurements available and has the added distinction of covering, at one time, a very wide range of gust wavelengths, the range of gust wavelengths covered being from 10 ft to about 50,000 ft. The result shown in the figure for the spectrum is given in three overlapping sections corresponding to the reduction procedure used for the several frequency ranges as given in Equations (6) to (8). The most significant aspect of the spectrum of the turbulence is the rapid decrease with increasing frequency of the spectral power. In fact, for frequencies greater than $\Omega = 0.001$, the frequency decreases approximately as Ω^{-2}, or perhaps within the accuracy of the data as $\Omega^{-5/3}$, a rate predicted by the theory of isotropic turbulence. At the lower frequencies, $\Omega < 0.001$, a tendency is indicated in these measurements, as in other studies, of a flattening of the spectrum. This region of frequencies has, however, so far received little attention.

In many analytical studies, the following analytic representation for the spectrum of vertical (or lateral) gust velocity has been used. The expression is given by

$$\Phi(\Omega) = \sigma_w^2 \frac{L}{\pi} \frac{1 + 3\Omega^2 L^2}{(1 + \Omega^2 L^2)^2} \tag{10}$$

where

σ_w = the root-mean-square gust velocity, ft/sec

L = the scale of turbulence, ft.

The quantity σ_w provides a simple measure of the magnitude of the spectrum or the intensity of the turbulence, while the scale of turbulence L provides a measure of the average eddy size and indicates the frequency at which the spectrum changes from a flat spectrum to one which decreases rapidly. This frequency is given approximately by $\Omega = 1/L$. The curve for $\sigma_w^2 = 33$ and L = 1,000 ft is shown in Figure 5 for comparison with the measurements and is seen to yield a good approximation to the measurements over almost the entire range of frequencies covered.

Except for differences in intensity, or in σ_w^2, the result shown in Figure 5 for the spectrum of vertical gust velocity appears to be representative of the measurements obtained in flight tests. However, it should be noted that the frequency ranges covered in most studies are quite limited. Also, the meteorological conditions covered so far in flight tests are limited and do not include measurements under strong convective clouds or at very low altitudes where conditions may give rise to a somewhat different spectral form.

3.2.3 Stationarity and Homogeneity

As indicated earlier, properties of stationarity and homogeneity of a random process specify an invariance in the statistical characteristics of turbulence with both starting time and starting position, respectively. Inasmuch as the intensity of turbulence and, in fact, its very existence in the atmosphere is dependent upon the broader weather conditions, these properties can only apply in a very limited sense. Weather conditions generally involve large-scale organized patterns of air motions which frequently cover hundreds of miles and which change relatively slowly with time; significant changes ordinarily take place in a matter of hours. As a consequence, homogeneous and stationarity conditions might at best only be expected to apply within limited regions of perhaps the order of 100 miles and time durations of perhaps the order of 1 hour. Some of the available data that have a bearing on these questions are presented as an indication of the applicability of these properties.

In Figure 6, a comparison is given of the power spectra of vertical gust velocity obtained from two successive sections of a flight record. These individual spectra each covered a flight distance of about 15 miles. The total record time covered by the two records is roughly 4 minutes. The frequency range covered in these tests is quite limited but covers the gust wavelengths of about 300 to 6,000 ft, which are the more important gust wavelengths for most airplane turbulence response problems. The two spectra agree in general form, considering that there are differences in both time and space involved in these records. However, the spectra also display a significant variation in the intensity of the turbulence at the lower frequencies, with the turbulence of part I, overall, being more severe than that for part II. One might surmise from this result that there is at least some degree of both homogeneity and stationarity present in the atmosphere at the higher frequencies, although variations do appear to exist even locally for the lower frequencies.

3.2.4 Gaussianness

The next property to be considered is that of the Gaussianness of atmospheric turbulence. As indicated in the Appendix, the condition of Gaussianness implies that the disturbance fluctuations have a Gaussian probability distribution. Furthermore, a Gaussian random process is one in which the joint distribution of the velocity fluctuations at several (all) points has a joint Gaussian distribution. The rigorous testing of experimental data for these conditions is by no means a simple undertaking. For present purposes, it will suffice to consider merely the overall probability distributions of the velocity fluctuations. This limited consideration is by no means conclusive but is generally a satisfactory practical guide.

A number of such evaluations have been made in the literature (see, for example, Reference 23). Some recent results obtained by the N.A.C.A. on this question are representative of these results and are shown in Figure 7. The figure shows the cumulative frequency distribution of gust velocity plotted on probability paper. The paper is scaled in a manner that yields a straight line for a normal or Gaussian probability distribution. The data plotted were based on 1/10-sec readings for a 4-min test run. For comparison with the measured distribution, a fitted normal distribution is also shown. It is readily seen that the fitted curve approximates the measured distribution quite well although some discrepancy is evident. Other comparisons of the same type indicate similar results although, on some occasions, the measured distributions appear to depart significantly for the extreme values from what would be expected for a Gaussian distribution. However, for many practical purposes, the Gaussian approximation appears quite adequate and reasonable.

3.2.5 Isotropy

As mentioned earlier, isotropy places a number of important restrictions on the allowable turbulence variations. The first property associated with isotropy is the property of the invariance of the statistical characteristics of turbulence with airplane flight direction. Figure 8 shows some results obtained in Reference 11 on the spectrum of vertical gust velocity as measured in two successive flight runs in an upwind and a crosswind flight path. These measurements unfortunately do not cover the whole frequency region of interest but are restricted to the higher frequencies covering gust wave lengths of 10 ft to several hundred ft. However, for this region of gust wavelengths, the spectra are in very good agreement, suggesting that, at least for the frequencies covered, the condition of isotropy was well approximated.

A second property of isotropy turbulence which is important is the specific relations required between the statistical characteristics of various turbulence components. As an illustration of the characteristics required, Figure 9 shows the lateral (or vertical) power spectrum of gust velocity defined by Equation 10 as well as the associated spectrum for the longitudinal gust velocity specified by Equation (24) in the Appendix for the use of isotropic turbulence. From Appendix Equation (24), it may be shown that the longitudinal component designated in the Appendix by $F(\Omega)$ is, for this case, given by

$$\Phi_v(\Omega) = \sigma_v^{\,2} \, \frac{2L}{\pi} \, \frac{1}{1 + \Omega^2 L^2}$$

For isotropic turbulence, both the vertical and side component of the turbulence sensed by an airplane would be expected to have the same spectrum. The longitudinal or head-on component of turbulence, while having the same mean-square-gust velocity, does, however, have a different spectral form such as indicated for the special case by the foregoing expression and as illustrated in Figure 9. As a consequence of Equation (24) of the Appendix, measurements of a single component of turbulence can be used to determine the spectra of the other components for the case of isotropic turbulence. In addition, for the case of isotropic turbulence, the measurement of a single power spectrum also permits the determination of cross spectra between directional components at any two points. The required relations may be obtained from Equation (22) of the Appendix and are, for example, given in Reference 26.

A few experimental results on the question of isotropy are shown in Figures 10 and 11. In Figure 10, a comparison is shown of the power spectra of the vertical and the side (or lateral) components of turbulence as obtained in Reference 11. The agreement between these two power spectra is quite good over the frequency region covered although the lateral or side component appears more severe than the vertical component of the turbulence. As in the previous case, the full region of frequencies of interest is not covered, with the data being restricted to the higher frequencies only. Some additional data recently obtained at the N.A.C.A. are shown in Figure 11. The curves shown here represent the spectra obtained for the vertical and the longitudinal turbulence components. The longitudinal spectrum was obtained from measurements of a sensitive and high-frequency airspeed system. Corrections for airplane longitudinal acceleration were not made. As indicated earlier, for isotropic turbulence, the spectrum of the vertical and longitudinal components should differ in the manner indicated in Figure 9. At the higher frequencies, there appears little difference between the two spectra of Figure 11. At the lower frequencies, the differences appear large, but these results are probably not very reliable in this frequency region.

As a matter of interest, it might be mentioned that efforts were also made to obtain the power spectrum of the lateral (or side) gust component for the test data covered in Figure 11. Preliminary results obtained appeared to indicate that at the lower frequencies the lateral turbulence was substantially more severe than the vertical. The reliability of this set of measurements has, however, not yet been established.

It appears from the foregoing that at best, the condition of isotropy may be expected to apply to atmospheric turbulence to a limited degree. In particular, the available information suggests that isotropy may only be approximated at the higher frequencies. At the lower frequencies, anisotropic conditions appear to be indicated by at least some of the measurements available.

3.2.6 *Limitations*

Mention should be made of the limitations present in the data so far obtained. For the most part, the turbulence measurements have been restricted to limited frequency regions and to a few atmospheric conditions. Measurements were, in most cases, obtained under clear-air flight conditions at moderate altitudes from 1,000 to 5,000 ft above terrain. There is good reason to believe that under other atmospheric conditions, the turbulence characteristics might be quite different from those so far obtained. First, for very low altitude conditions, say below 1,000 ft, it might well be expected that turbulence, being frequently wind generated, would more closely reflect the local terrain characteristics. As a consequence, conditions of homogeneity would be largely dependent upon terrain homogeneity and conditions of stationarity would be dependent upon persistence of the broad wind field. The spectral characteristics for this case might also be expected to be more sensitive to terrain as well as the thermodynamic conditions. Finally, it is questionable whether conditions of isotropy might be approximated under such conditions and, in fact, some measurements now available from meteorological towers suggest that isotropy, while approximated at the higher frequencies, does not appear to extend over the entire region of frequencies that is of interest in aeronautical applications.

The second weather condition that deserves special comment is that of strong convective clouds or thunderstorms. Because of the severe conditions of turbulence under such conditions, this weather condition is of particular concern in aeronautical applications. The basic physical element of a thunderstorm is normally of small dimension, of the order of perhaps 5 to 10 miles in height and width, and contains a well-organized large-scale flow pattern which generates smaller eddies or turbulent velocity fluctuations[29]. As a consequence, it might be expected that the turbulence would more closely reflect the larger scale thunderstorm structure and turbulence measurements obtained under other conditions, such as in clear air, may not be representative of conditions for this case.

4. PROBABILITY DISTRIBUTIONS OF ROOT-MEAN-SQUARE GUST VELOCITY

In the preceding section, it was indicated that for given conditions of time and place, atmospheric turbulence might be considered to approximate a homogeneous and Gaussian random process. Over most of the frequency region of concern, the spectral form of the vertical (or lateral) gust velocity appeared to be approximated by the following expression (Equ. 10)

$$\Phi(\Omega) = \sigma_w{}^2 \frac{L}{\pi} \frac{1 + 3\Omega^2L^2}{(1 + \Omega^2L^2)^2}$$

with values of the scale of turbulence L of the order of 1,000 ft. However, the values of the root-mean-square gust velocity σ_w appear to vary widely with weather condition. In this section of the present paper, this simplified model of the local turbulence will be used as a building block to provide a more general description of the overall turbulence history experienced by airplanes in operational flight. Such a description is desirable and needed in order to permit the determination of the airplane loads and other response histories in operational flight.

The detailed measurements of turbulence and turbulence spectra discussed in the last section are still far too meager to supply the information required. Direct measurements of turbulence power spectra to provide what might be called a turbulence climatology appears to be a distant future possibility. A vast body of statistical data from normal operations on airplane acceleration responses in rough air does exist, however. These data have, in most cases, been collected in terms of the total number of peak accelerations (or effective gust velocities) that exceeded given values of acceleration (or gust velocity) in operational flights. Under certain simplifying assumptions, it has appeared possible to use these data to derive reasonable estimates of the turbulence climatology of the atmosphere in the form of the probability distributions of the root-mean-square vertical gust velocity for various conditions. Such results are reported in References 18, 19 and 25. Inasmuch as these results provide at least a beginning of a description of turbulence that can be used in spectral-type calculations of operational response histories, it appeared appropriate to review briefly the general approach used and some of the results obtained.

4.1 Method of Analysis

The general method used in these studies involves two steps. These are:

(1) the derivation of the probability distribution of the root-mean-square accelerations from the data on counts of acceleration peaks

(2) the transformation of the root-mean-square acceleration to root-mean-square gust velocities through the use of the airplane response characteristics.

These two steps will be discussed in order.

4.1.1 Derivation of the Probability Distribution of Root-Mean-Square Acceleration

For a stationary Gaussian random process, say y(t), a simple approximate relation exists between the number of peaks per second exceeding a given value of y and the power spectrum of y and is derived in Reference 26. This relation is also discussed in References 18 and 25 and is given by

$$N(y) = N_0 e^{-y^2/2\sigma^2} \tag{11}$$

where σ^2 equal to the mean square value of y and N_0 can be considered a characteristic frequency of y(t) and is defined by

$$N_0 = \frac{1}{2\pi}\left[\frac{\int_0^\infty \omega^2 \Phi_y(\omega)\,d\omega}{\int_0^\infty \Phi_y(\omega)\,d\omega}\right]^{\frac{1}{2}} \tag{11a}$$

In Reference 18, this relation is extended to the case of an airplane flying through continuous rough air of fixed spectral form but variable root-mean-square gust velocity. For this case, the average number of peak accelerations per second exceeding a given value is given by

$$\overline{M(a_n)} = N_0 \int_0^\infty \left[p(\sigma_a) e^{-a_n{}^2/2\sigma_a{}^2} \right] d\sigma_a \tag{12}$$

where

$\overline{M(a_n)}$ = the average number of peak accelerations per second exceeding a given value of a_n

σ_a = root-mean-square value of a_n

$p(\sigma_a)$ = the probability distribution of σ_a.

Equation 12 permits the determination of the probability distribution of the root-mean-square acceleration $p(\sigma_a)$ from measured distribution of acceleration peaks $\overline{M(a_n)}$. In most cases, the integral Equation (12) cannot be solved in closed form, but graphical or numerical estimates of $p(\sigma_a)$ can be readily derived (see Reference 25). (This same approach may also be applied if the measurements are in terms of counts of effective gust velocities by considering the effective gust velocities as reduced acceleration values.)

4.1.2 Probability Distributions of Root-Mean-Square Gust Velocity

The conversion of the derived distributions of root-mean-square acceleration to those for root-mean-square gust velocity σ_w requires the determination of the relation between the two root-mean-square values. If the airplane is assumed a linear system and average values of such airplane characteristics as weight, air-speed, air-density, center-of-gravity position, etc., are taken as adequate, then for a given operation or portion of an operation, the root-mean-square values are related simply as follows

$$\sigma_a = \bar{A}\sigma_w \tag{13}$$

where

$$\bar{A} = \frac{1}{\sigma_w}\left[\int_0^\infty \Phi_{,}(\omega)\,|H(\omega)|^2 d\omega\right]^{\frac{1}{2}} \tag{14}$$

For the case of a rigid airplane free to move vertically but not pitch, the quantity $\bar{A}$ is given by

$$\bar{A} = \frac{\rho VSm}{2W}\sqrt{\frac{I(K,s)}{\pi}} \tag{15}$$

where

ρ = air density

V = forward speed

S = wing area

m = slope of the lift curve

W = airplane weight

I(K,s) = an airplane gust response function (defined in Reference 4) which depends upon the airplane mass parameter $K = 4W/g\pi\rho S\bar{c}$ and $s = \bar{c}/L$ where $\bar{c}$ is the mean wing chord.

This case is used in References 18, 19, and 25.

If the relation between root-mean-square values of Equation (13) is assumed to apply, the probability distribution of root-mean-square gust velocity is then given by

$$\hat{p}(\sigma_w) = \bar{A}p(\bar{A}\sigma_a) \tag{16}$$

Equation (16) permits the simple conversion of probability distributions of root-mean-square accelerations to root-mean-square gust velocities. This conversion step is, for the case considered, seen to depend only upon the response quantity $\bar{A}$.

4.2 Application to Gust Load Data

In this section, some results obtained by the application of the relations derived in the preceding section to available statistical data on gust loads are presented. Three principal problems are encountered in such applications. These involve the choice of an appropriate value of the scale of turbulence L (Eq. 10), the determination of the airplane acceleration frequency response function for gusts $H(\omega)$ (or more particularly, the value of $\bar{A}$), and the determination of the value of the characteristic frequency N_0. The value of L chosen for present purposes was 1,000 ft (although other values were also considered in Reference 18) which, on the basis of available data, appears to be a reasonable estimate for an average value for atmospheric turbulence. In the determination of the airplane frequency response function and the values of $\bar{A}$ for the various airplanes involved, the airplane accelerations are assumed entirely due to the vertical gusts and the airplane is assumed to respond in vertical motion only (no pitch, rigid-body condition). A justification for this restriction to the one-degree-of-freedom response for this purpose is given in Reference 25. Finally, it was found simple and expedient to obtain good estimates of the characteristic frequency N_0 directly from the flight records of acceleration for the various airplanes in accordance with the methods given in Reference 25.

4.2.1 *Variation of $\hat{p}(\sigma_w)$ with Altitude*

In order to arrive at some estimates of the variation of $\hat{p}(\sigma_w)$ with altitude, use was made of the summary of gust statistics given in Reference 31. Figure 6 of Reference 31 presents estimates of the average gust experience at various altitudes that are representative of contemporary types of transport operations. In order to estimate the associated distributions of root-mean-square gust velocity, these results, which are in terms of derived gust velocities, were, for convenience, first converted to accelerations by using the characteristics of a representative transport airplane. The charts of Reference 25 were then used to estimate the appropriate probability distribution for the root-mean-square gust velocity. It was found that, for each altitude bracket, the number of peak accelerations for the various altitude brackets could be approximated by the following distributions for $\hat{p}(\sigma_w)$:

for the altitude bracket of 0 to 10,000 ft:

$$\hat{p}(\sigma_w) = 0.99\frac{1}{1.48} e^{-\sigma_w/1.48} + 0.01 \frac{1}{2.48} e^{-\sigma_w/2.84} \tag{17}$$

for the altitude bracket 10,000 to 30,000 ft:

$$\hat{p}(\sigma_w) = \frac{1}{2(0.32)^2} e^{-\sqrt{\sigma_w}/0.32} \tag{18}$$

for the altitude bracket 40,000 to 50,000 ft:

$$\hat{p}(\sigma_w) = \frac{1}{2(0.29)^2} e^{-\sqrt{\sigma_w}/0.29} \tag{19}$$

These distributions are shown in Figure 12 in terms of their cumulative probability distributions $\hat{P}(\sigma_w)$ (obtained by integrating Eqs. 17 to 19 from given values of σ_w to infinity) and indicate the proportion of flight time spent above given values of σ_w for the various altitude brackets. Perhaps the most important points to be noted in Figure 12 are the relatively large amount of time spent in essentially smooth air at the higher altitudes (σ_w < 2 ft/sec, 93 to 96% of the time) and the relatively large amount of time spent in light to severe turbulence at the lowest altitude bracket (σ_w > 2 ft/sec, 25% of the time). The total time spent for σ_w > 5 ft/sec for the lowest altitude bracket is roughly 5 to 10 times as great as that for the higher altitude brackets.

4.2.2 Method of Application

The foregoing results may be applied in a straightforward manner to the calculation of response histories for arbitrary operations. The method of application of these results to the calculations of response histories is based on the following relation:

$$\overline{M(a_n)} = N_0 \int_0^\infty \left[\hat{p}(\sigma_w) e^{-a_n^2/2(\bar{A}\sigma_w)^2} \right] d\sigma_w \qquad (20)$$

which is obtained by substituting Equations 13 and 16 into Equation 12. The procedure involves the division of the operational flight plan into homogeneous portions or segments in regard to flight altitude and operating conditions such as airspeed and airplane weight. The appropriate distribution of the root-mean-square gust velocity is selected for each flight segment from Figure 12 and Equation 20 is then evaluated for each segment. Actual numerical calculations are facilitated by the use of the charts given in References 18 and 25. The sequence of steps involved in such applications is as follows:

(1) The operational flight plan is divided into homogeneous segments in regard to flight altitude (10,000 ft altitude brackets) and operating conditions such as airspeed and weight;

(2) The appropriate distribution of $\hat{p}(\sigma_w)$ is selected for each altitude bracket from Figure 12;

(3) The values of $\bar{A}$ for the acceleration or other response are determined for each significant segment of the flight plan in accordance with Equation (14);

(4) In order to obtain the associated distributions of root-mean-square acceleration $p(\sigma_a)$, each of the distributions of $\hat{p}(\sigma_w)$ is transformed by the relation

$$p(\sigma_a) = \frac{1}{\bar{A}} \hat{p}(\sigma_w)$$

where

$$\sigma_w = \frac{\sigma_a}{\bar{A}}$$

(5) The values of N_0 are most easily determined from flight records, if available, by the methods already indicated[25]. For new designs N_0 must be estimated analytically by application of Equation (11a). In terms of the power spectrum of the gust input, N_0 is given by

$$N_0 = \frac{1}{2\pi}\left[\frac{\int_0^\infty \omega^2\Phi_w(\omega)\,|H(\omega)|^2 d\omega}{\int_0^\infty \Phi_w(\omega)\,|H(\omega)|^2 d\omega}\right]^{\frac{1}{2}}$$

(6) The distribution of σ_a and the values of N_0 are then used in Equation (12) to derive the number of peak accelerations per second or per mile for each condition. These calculations are facilitated by the use of charts such as given in Reference (18) or (25).

(7) The results obtained in step 6 are then weighted in accordance with the flight distance in each condition or segment and then summed for all conditions in order to obtain the overall acceleration or other response history.

5. CONCLUDING REMARKS

The foregoing discussion has served to provide a preliminary assessment of the characteristics of continuous atmospheric turbulence as determined from recent flight-test measurements. In the discussion, the general methods that have been used for obtaining measurements of continuous atmospheric turbulence have been reviewed and some of their limitations indicated. In spite of the limitations, the techniques used, and particularly the technique based on flow direction measurements, appear to provide a reliable measurement of continuous atmospheric turbulence. In evaluating the measurements so far obtained, the local characteristics of turbulence were considered, and particular attention was given to the properties of the power spectrum of turbulence. For the measurements so far obtained, the power spectrum appears to show a persistent spectral form over a wide range of frequencies and, in particular, appears to decrease with increasing frequency as $1/\Omega^2$ for the range of frequencies of principal concern.

The following analytic expression

$$\Phi_w(\Omega) = \sigma_w{}^2 \frac{L}{\pi} \frac{1 + 3\Omega^2 L^2}{(1 + \Omega^2 L^2)^2}$$

where

σ_w = root-mean-square gust velocity, and

L = scale of turbulence,

appears to represent the available measurements of the spectrum of vertical gust velocity reasonably well. For the available measurements, the value of L appears to be about 1,000 ft, while values of σ_w vary over a wide range.

In addition to the spectral properties, the characteristics of the measurements were also examined in regard to properties frequently used in idealized and simplified theoretical models. In particular, the properties of stationarity, homogeneity, Gaussianness, and isotropy were considered and the measurements examined to determine how well these properties apply to the atmosphere. Examination of the measurements available suggests that these properties appear to apply to atmospheric turbulence

to a limited degree. The applicability of most of these properties appears best at the highest frequencies and poorest at the lower frequencies. The evidence thus suggests that while analytical studies based on such simple models may prove useful guides, they must be applied with caution. Further measurements of atmospheric turbulence are needed in order to cover both a wider range of frequencies and a wider range of meteorological conditions. In particular, a need for measurements under such important conditions as very low altitudes and strong convective clouds or thunderstorms is indicated.

In addition to the consideration of the local characteristics of atmospheric turbulence, consideration was also given to the problem of applying the techniques of continuous turbulence analysis to the calculation of airplane response in operational flight. For this purpose, recent results obtained on the probability distributions of the root-mean-square gust velocities were described and their method of application to response calculations indicated.

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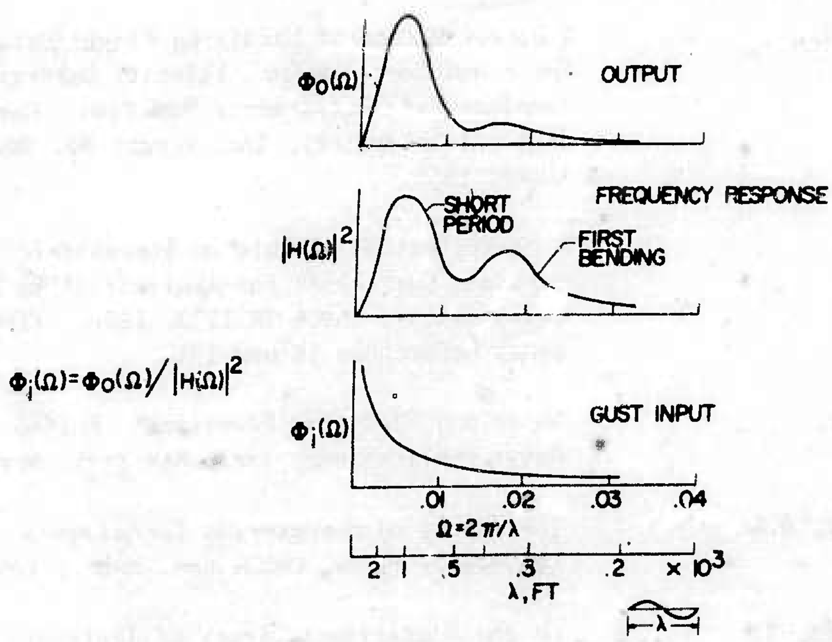
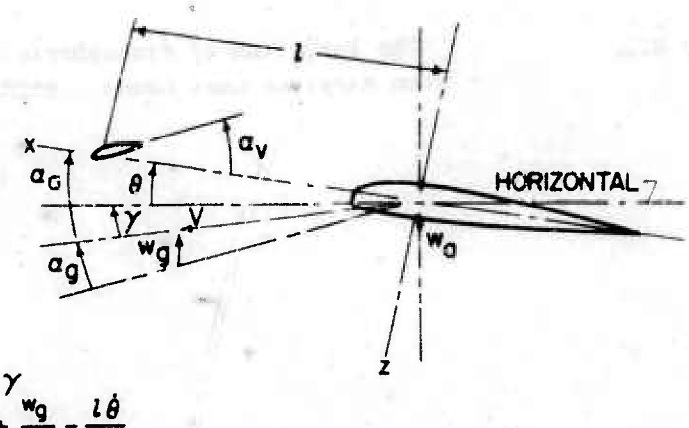


Fig. 1 Method of determination of the power spectrum of gust velocity from airplane response measurements



(1) $a_a = \theta + \gamma$

(2) $a_v = a_a + \frac{w_g}{V} - \frac{l\dot{\theta}}{V}$

(3) $w_g = V(a_v - \theta) - w_a + l\dot{\theta}$

Fig. 2 Geometric relations between vertical gust velocity and vane angle

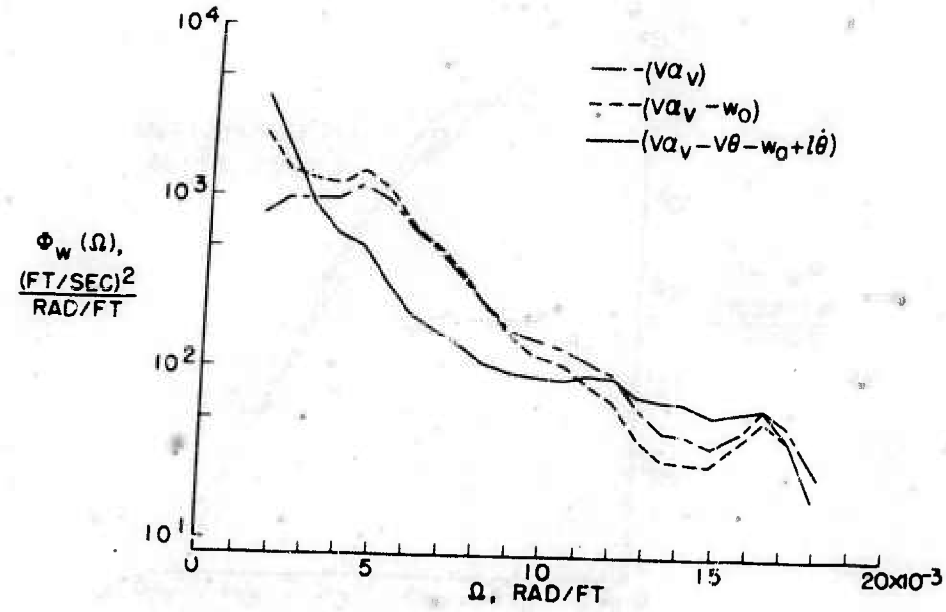


Fig. 3 Comparison of power spectrum of gust velocity as obtained with and without corrections for airplane pitching and vertical velocity motions

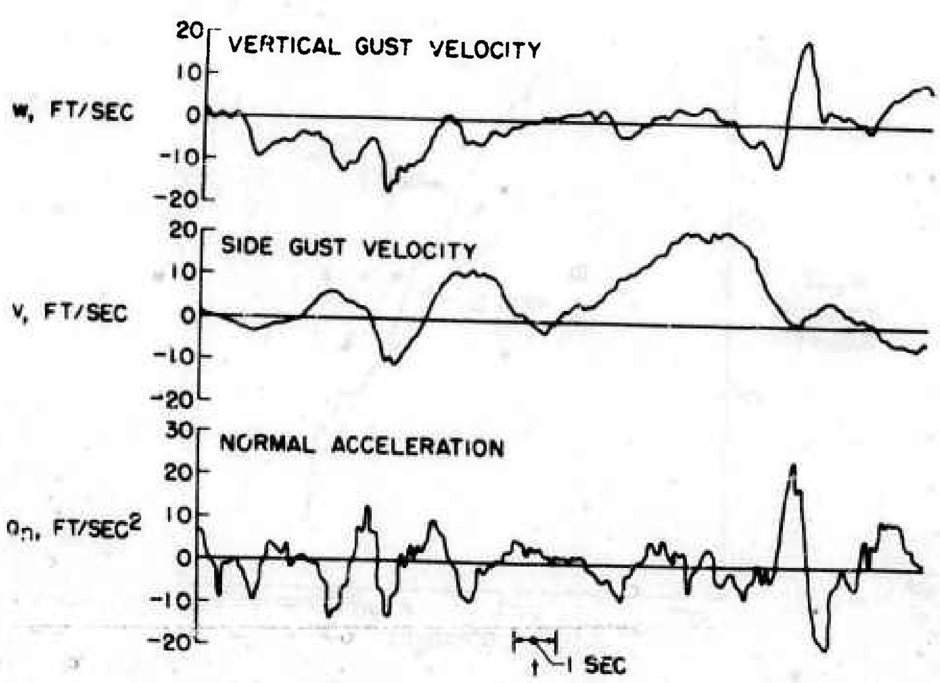


Fig. 4 Comparison of time history of vertical gust velocity, side gust velocity, and airplane normal acceleration

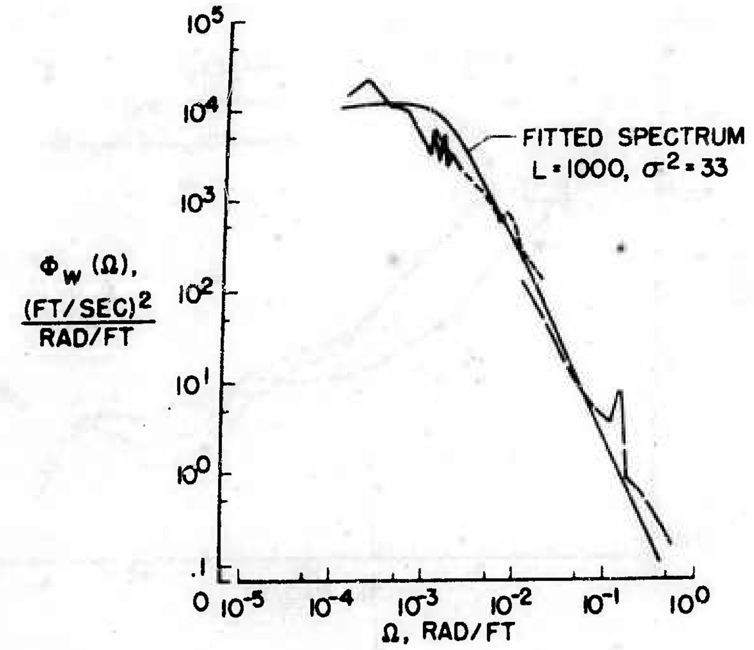


Fig. 5 Measured power spectrum of vertical gust velocity and analytic representation

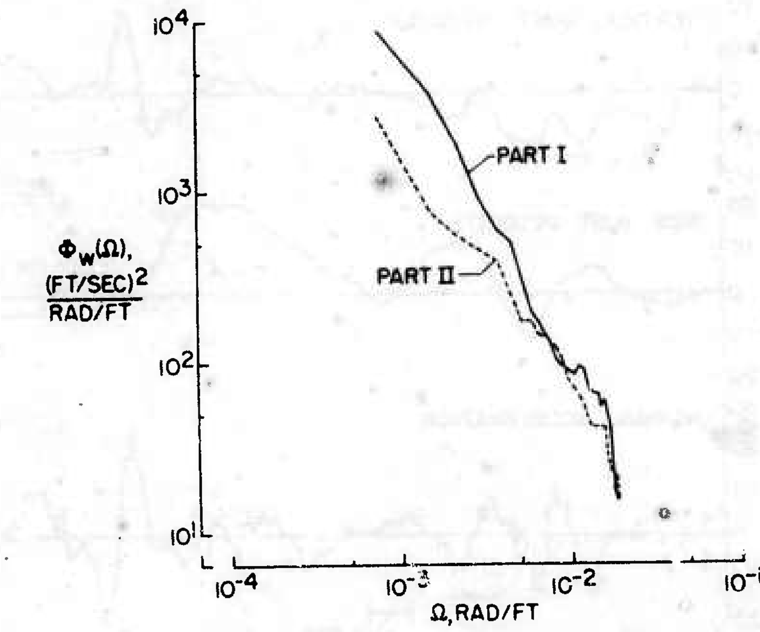


Fig. 6 Comparison of power spectra of vertical gust velocity for two parts of a test run

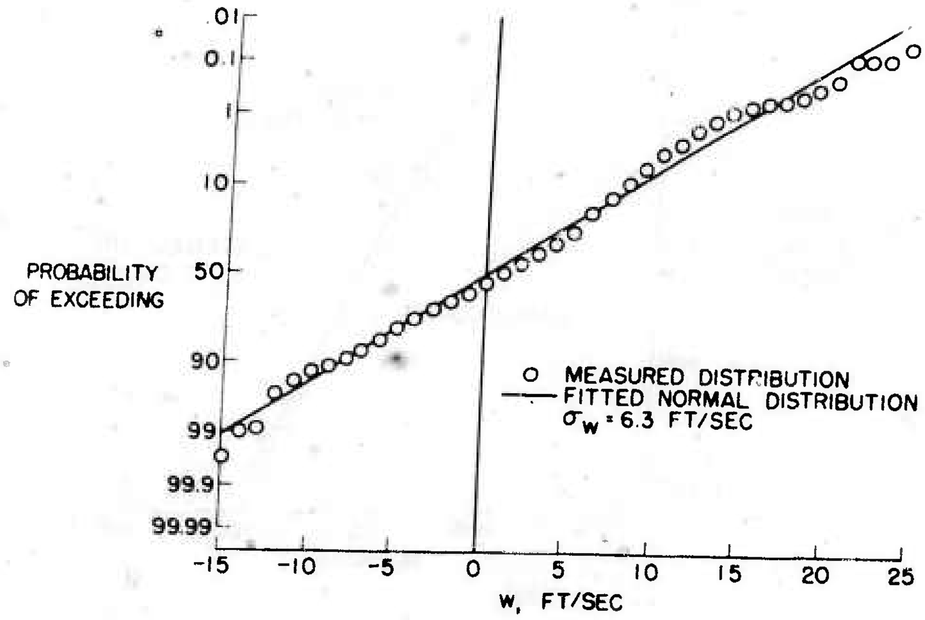


Fig. 7 Probability of equalling or exceeding given values of w_g

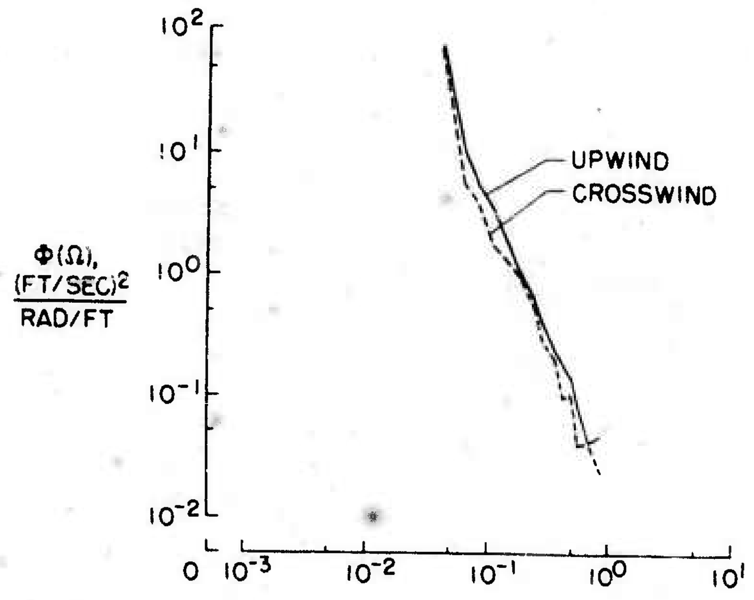


Fig. 8 Comparison of the power spectra of vertical gust velocity for upwind and crosswind flight surveys (Ref. 11)

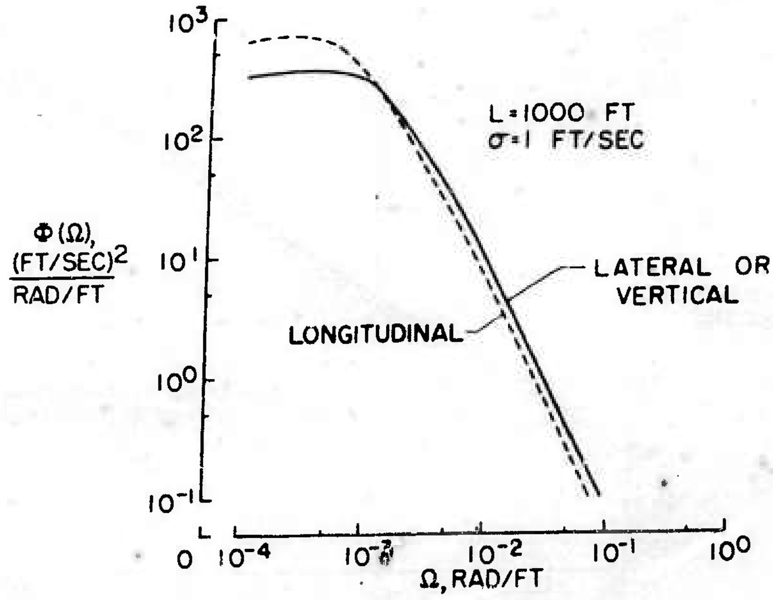


Fig. 9 Comparison of lateral (or vertical gust velocity power spectrum of Equation 10 in the main text with longitudinal spectrum for case of isotropic turbulence

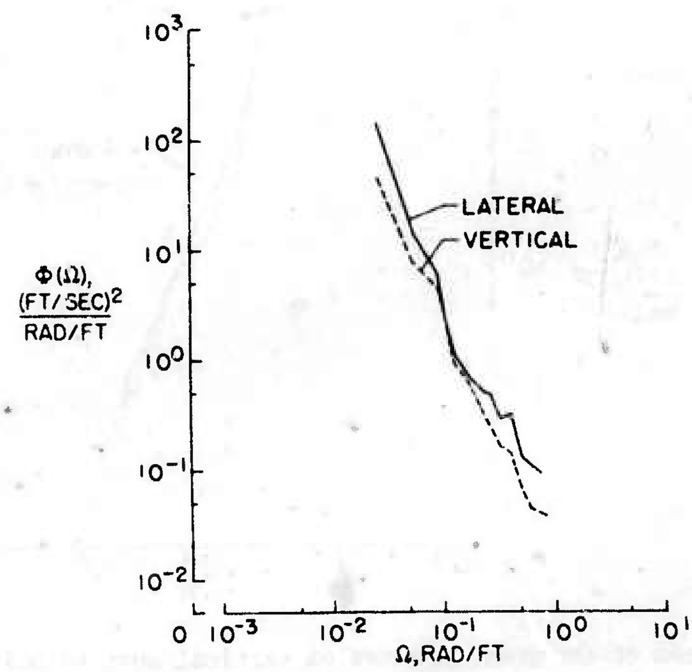


Fig. 10 Comparison of the power spectra of lateral and vertical gust velocity (Ref. 11)

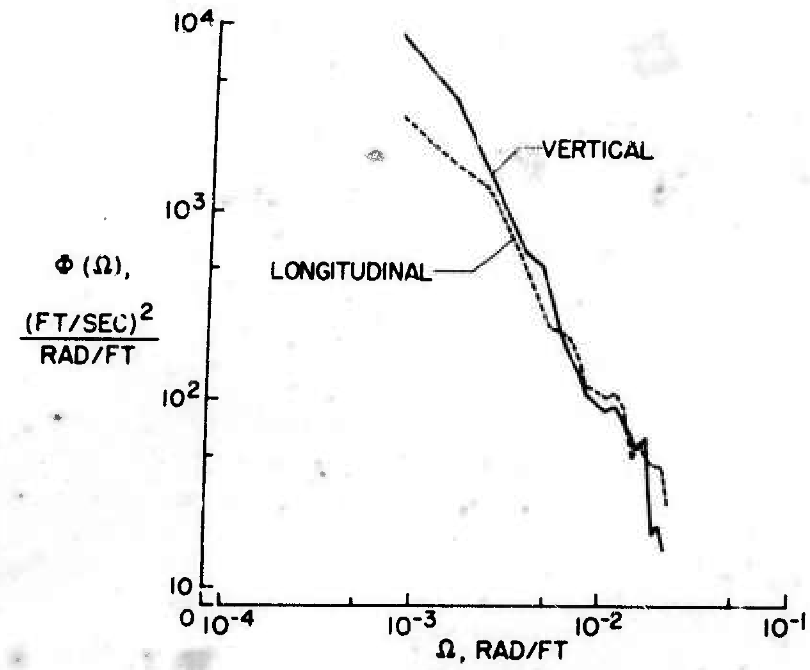


Fig. 11 Comparison of the power spectra of vertical and longitudinal gust velocity

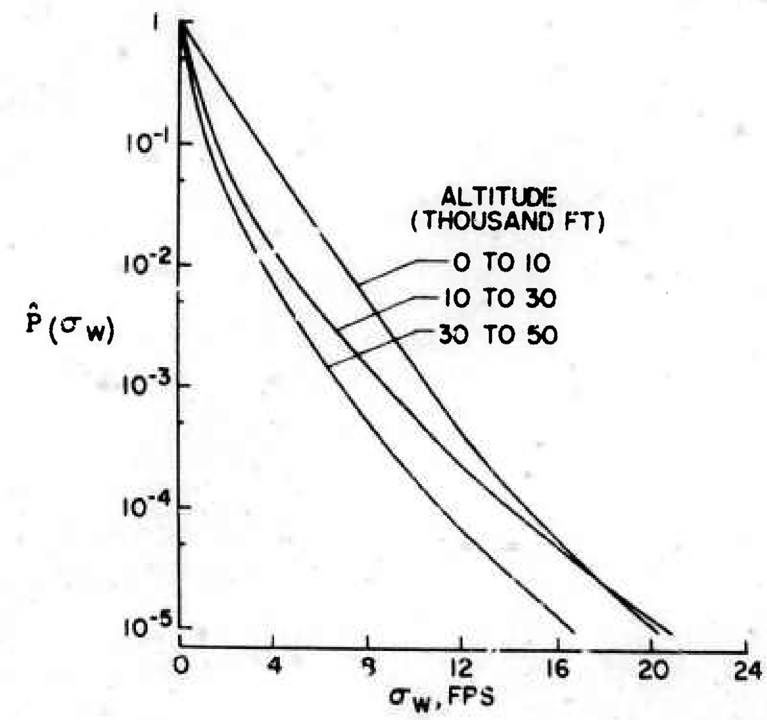


Fig. 12 Cumulative probability distributions of root-mean-square gust velocity for various altitudes

APPENDIX

FUNDAMENTAL CONCEPTS OF RANDOM PROCESS THEORY

APPENDIX

Fundamental Concepts of Random Process Theory

1. GENERAL REMARKS

In this appendix, some of the basic concepts and relations of random process theory and the statistical theories of turbulence pertinent to the analysis and interpretation of measurements of atmospheric turbulence will be reviewed. Rather complete treatments of these two areas, random process theory and the statistical theories of turbulence, are contained in References 30 and 27, respectively. The present discussion is aimed at defining some of the specialized terminology and indicating a few of the more important relations that are required for present purposes. The discussion will start with the consideration of a simple random process $f(x)$ involving the single parameter x. The basic concept of a power spectrum of a random process and the important case of the Gaussian random process will be described. An indication will also be given of the methods of application of these ideas to airplane response problems.

In many aeronautical applications, generalizations of the foregoing simple cases are required. For example, recent studies[12,22] have indicated that in addition to variations of turbulence along the flight path, the variations in turbulence in the airplane spanwise direction must frequently be taken into account in determining both airplane lateral and longitudinal responses. The consideration of the turbulence variations in these two directions requires an extension to two-dimensional processes $f(x,y)$. The most general case that might be considered is a four-dimensional process $f(x,y,z,t)$; the four-dimensional process accounting for the variations in all three space coordinates as well as time. However, in practice, this full generalization is rarely required and would unduly complicate the present discussion. Turbulence variations with time can frequently be neglected on the basis that the turbulence pattern changes slowly with respect to the rapid forward speed of the airplane. This assumption is the equivalent of Taylor's hypothesis, which is commonly applied in studies of wind-tunnel turbulence. Thus, in aeronautical application, the airplane is generally envisioned as sampling an instantaneous cross section of the turbulence in space. The variations of turbulence in the z direction may also usually be ignored since in most cases the airplane motions in a direction normal to the average flight path are considered small.

In addition to the generalization to multidimensional random processes, a further generalization to vector processes is frequently required in order to account for the directional components of the turbulence. For example, in regard to the airplane normal response, an adequate analysis sometimes requires the consideration of both the normal and the longitudinal or head-on components of atmospheric turbulence. Thus, in the general case, the process $f(x,y)$ has to be considered as a vector process with three normal directional components u, v, and w. The general consideration of multidimensional vector processes rapidly becomes too involved for many practical applications. As a consequence, the necessity exists for simplifications if practical results are to be achieved. Fortunately, simplifications can frequently be made involving symmetries in the space coordinates and internal relationships between the statistical characteristics of the turbulence directional components. The most important and useful of these simplifications is the case of isotropic turbulence, which will also be considered.

2. SINGLE DIMENSIONAL RANDOM PROCESSES

2.1 Random Processes

The term random immediately implies a lack of definiteness that can only be described in statistical or probability terms. In the general sense, a random process may be said to consist of a set of functions (ensemble), say $f(x)$, where the parameter x may, for example, designate time or as in the present case a space coordinate, and where the individual functions $f_a(x)$, $f_b(x)$, etc., each occurs with a given probability. Alternately, the process $f(x)$ may be defined in terms of probability distributions for given values of x. The function $p(f_1)$ is used to designate the probability distribution of $f(x)$ at position x_1 and the joint probability distribution of f for various positions, x_1, x_2, etc., may be designated by

$$p(f_1, f_2, \ldots)$$

where the subscripts designate the values of f for various values of the parameter x. The probability distribution may be said to define the proportion of all members of this set that have given combinations of values at the various positions, and is thus taken to define the statistical characteristics of the process completely.

The definition of a random process in terms of its complete probability distribution is, however, generally too cumbersome for practical applications and recourse is generally had to simpler and more manageable properties of the process. Of these, the most important are the average values

$$\left.\begin{aligned} \text{Av.}\{f(x)\} &= \int_{-\infty}^{\infty} f p(f) df \\ \text{Av.}\{f^2(x)\} &= \int_{-\infty}^{\infty} f^2 p(f) df \\ \text{Av.}\{f(x_1)\, f(x_2)\} &= \int_{-\infty}^{\infty} f_1 f_2 p(f_1, f_2) df_1\, df_2 \\ &= R_{ff}(x_1, x_2) \end{aligned}\right\} \quad \text{(A.1)}$$

These three averages are sufficient and most useful for many purposes. The first expression defines the average value for various values of x. The second quantity, $\text{Av.}\{f^2(x)\}$, defines the mean-square value, and the quantity $\text{Av.}\{f(x_1)\, f(x_2)\}$, defines the average product of f_1 and f_2. In the special case of a stationary Gaussian random process, which is of particular interest in present considerations, these quantities will be seen to specify the statistical characteristics of the process completely.

2.2 Stationarity and Homogeneity

As defined in Equation (A.1) herewith, the average values are clearly functions of the parameter x. In many practical applications, it is useful to consider the special case in which these averages are independent of the value of x. If the quantity x designates time, such processes are termed stationary and the average values and probability distributions do not depend upon the starting point x_1.

Alternatively, if the parameter is a space parameter, as is generally the case in turbulence applications, the process is termed homogeneous. Under such conditions

$$\left.\begin{aligned} \text{Av.}\{f(x_i)\} &= \text{Av.}\{f(x_1)\} \\ \text{Av.}\{f^2(x_i)\} &= \text{Av.}\{f^2(x_1)\} \\ \text{Av.}\{f(x_i)\ f(x_j)\} &= R_{ff}(x_j - x_i) \end{aligned}\right\} \quad (A.2)$$

Under certain conditions (which generally apply in practice), stationary processes have the important and useful ergodic property of the equivalence of averages over the set of functions with averages in time (or in space) for a single function. Thus we may write that

$$\left.\begin{aligned} \text{Av.}\{f(x)\} &= \lim_{X\to\infty} \frac{1}{2X}\int_{-X}^{X} f(x)dx \\ \text{Av.}\{f^2(x)\} &= \lim_{X\to\infty} \frac{1}{2X}\int_{-X}^{X} f^2(x)dx \\ \text{Av.}\{f(x_i)\ f(x_j)\} &= \lim_{X\to\infty} \frac{1}{2X}\int_{-X}^{X} f(x)\ f(x+\Delta x)dx = R_{ff}(\Delta x) \end{aligned}\right\} \quad (A.3)$$

where $\Delta x = x_j - x_i$. Thus, a single member of the set, which is frequently all that is available in practice, may be used to define the statistical properties of the entire set. It will, without loss of generality, be assumed that the mean value defined by Av.$\{f(x)\}$ is equal to zero. For this case, the second moment average Av.$\{f^2(x)\}$ is generally termed the variance or mean square or power and the quantity $R_{ff}(\Delta x)$ is termed the auto-covariance function or the auto-correlation function. The auto-correlation function is symmetrical about the origin and has its maximum value at $\Delta x = 0$.

2.3 Power Spectra

The auto-correlation function is particularly important since it reflects the frequency characteristics of the disturbance, and its Fourier transform gives rise to the power spectrum of the random process. The power spectrum of f(x) is defined by the reciprocal relations

$$\left.\begin{aligned} \Phi_{ff}(\Omega) &= \frac{1}{\pi}\int_{\infty}^{\infty}\left[R_{ff}(\Delta x)e^{-i\Omega\Delta x}\right]d\Delta x \\ R_{ff}(\Delta x) &= \frac{1}{2}\int_{-\infty}^{\infty}\left[\Phi(\Omega)e^{i\Omega\Delta x}\right]d\Omega \end{aligned}\right\} \quad (A.4)$$

where

Ω = reduced frequency, radians/ft, and

Δx = space displacement, ft.

The power spectrum is a real and positive symmetrical function because of the symmetry of $R_{ff}(\Delta x)$ and is a particularly useful function since it describes the distributions of the energy of $f^2(x)$ with frequency. Because of the symmetry of $\Phi_{ff}(x)$ and $R_{ff}(\Delta x)$, Equation (A.4) herewith may be simplified to reciprocal cosine transformations.

2.4 Cross Spectra

For the case of two random processes, $f(x)$ and $g(x)$, additional averages of the form

$$\text{Av.}\{f(x_i)\ g(x_j)\} = R_{fg}(x_j - x_i)$$

are also of interest. Such averages reflect the correlation between the two processes and thus $R_{fg}(x_j - x_i)$ is called the cross-correlation function. For stationary and ergodic processes

$$\text{Av.}\{f(x_i)\ g(x_j)\} = \lim_{X\to\infty} \frac{1}{2X}\int_{-X}^{X} f(x)\ g(x + \Delta x)d\Delta x \qquad \text{(A.5)}$$

The cross spectrum is not symmetrical and is easily shown to have the property that

$$R_{fg}(\Delta x) = R_{gf}(-\Delta x) \qquad \text{(A.6)}$$

The Fourier transform of $R_{fg}(\Delta x)$ may also be defined and yields the so-called cross spectrum $\Phi_{fg}(\Omega)$ between $f(x)$ and $g(x)$. The reciprocal relations between $R_{fg}(\Delta x)$ and $\Phi_{fg}(\Omega)$ are given by

$$\left.\begin{aligned} \Phi_{fg}(\Omega) &= \frac{1}{\pi}\int_{-\infty}^{\infty}\left[R_{fg}(\Delta x)e^{-i\Omega\Delta x}\right]d\Delta x \\ &= c(\Omega) - iq(\Omega) \\ R_{fg}(\Delta x) &= \frac{1}{2}\int_{-\infty}^{\infty}\left[\Phi_{fg}(\Omega)e^{i\Omega\Delta x}\right]d\Omega \end{aligned}\right\} \qquad \text{(A.7)}$$

The cross spectrum, in contrast to the power spectrum, is complex, having both real and imaginary parts. The real part $c(\Omega)$ is generally termed the cospectrum and provides a measure of the in-phase power of the two disturbances. The imaginary part $q(\Omega)$ is generally termed the quadrature power and provides a measure of the 90^0 out-of-phase power between the two disturbances. From Equation (A.6), it may be shown that

$$\Phi_{fg}(\Omega) = \Phi_{gf}(-\Omega) \qquad \text{(A.8)}$$

2.5 Input-Output Relations

The power spectrum and cross spectrum have the important property of permitting linear operations in simple form. The response of an airplane, flying along the x axis at a given speed, to an arbitrary gust disturbance $f(x)$, may be expressed as

$$z(x) = \int_0^\infty f(x - x_1)\, h(x_1) dx_1 \qquad (A.9)$$

where h(x) is the response of the airplane to a unit impulse gust at x = 0 and is generally termed the unit impulse response function. The linear relation of Equation (A.9) leads to the following relations among the power spectrum of the response $\Phi_{zz}(\Omega)$ and the cross spectrum between the disturbance and the response $\Phi_{fz}(\Omega)$:

$$\Phi_{zz}(\Omega) = \Phi_{ff}(\Omega) |H(\Omega)|^2 \qquad (A.10)$$

$$\Phi_{fz}(\Omega) = \Phi_{ff}(\Omega) H(\Omega) \qquad (A.11)$$

where

$$H(\Omega) = \int_0^\infty \left[h(x) e^{-i\Omega x} \right] dx$$

and is generally termed the frequency response function and defines the system response to unit sinusoidal gusts of frequency Ω. Thus, it may be seen that the integral relation of Equation (A.9) yields simple product relations between the spectra. These simple product relations for linear systems provide a major reason for the usefulness of spectral methods.

2.6 Gaussian Random Process

The special case of stationary random process that is of particular interest because of its natural simplicities and because it frequently approximates the conditions of natural phenomena, is the case of a Gaussian process. In the present paper, it is of interest to see how well this model approximates the condition of atmospheric turbulence. This special case has the property that the probability distributions is Gaussian. Specifically, for any value of x

$$p(f) = \frac{1}{\sigma\sqrt{2\pi}} e^{-f^2/2\sigma^2} \qquad (A.12)$$

where $\sigma^2 = R_{ff}(0)$. The joint probability distribution for several values of x also has a multi-variate Gaussian distribution. Thus, for example, the joint distribution of f for x_1 and x_2 is given by

$$p(f_1, f_2) = \frac{1}{2\pi(\sigma^4 - R_{12}{}^2)^{\frac{1}{2}}} \exp - \left[\frac{\sigma^2 f_1{}^2 - 2R_{12} f_1 f_2 + \sigma^2 f_2{}^2}{2(\sigma^4 - R_{12})} \right] \qquad (A.13)$$

where $R_{12} = R_{ff}(x_2 - x_1)$.

This distribution, as well as higher-order distributions for x_1, x_2, x_3, etc., depends only upon the second-order averages of the random process given by the auto-covariance function $R_{ff}(\Delta x)$, and in this sense, the process is said to be completely specified by the auto-covariance function or its Fourier transform, the power spectral density function.

The characteristics of a Gaussian process have been investigated extensively and a number of useful relations have been derived for this type of process which permit the estimation of statistical quantities of frequent interest from the basic quantity,

the power spectrum. Many of these relations are given in Reference 26, and a few of the more important ones in aeronautical applications are given in Reference 20.

For a stationary Gaussian process, the ergodic property will generally apply, and the probability distributions obtained from a single function should also be Gaussian. Thus, a single time history sample can be examined to determine if the process is Gaussian.

3. MULTIDIMENSIONAL AND VECTOR PROCESSES

3.1 Two-Dimensional Power Spectra

In analogy with Equation (A.1) of this Appendix for the single-dimensional process, we may, for two-dimensional process $f(x,y)$ note that

$$\left.\begin{aligned} \text{Av.}\{f(x,y)\} &= \int_{-\infty}^{\infty} f p(f) df \\ \text{Av.}\{f^2(x,y)\} &= \int_{-\infty}^{\infty} f^2 p(f) df \\ \text{Av.}\{f(x_1,y_1)\ f(x_2,y_2)\} &= \int_{-\infty}^{\infty} f_1 f_2 p(f_1,f_2) df_1\ df_2 \end{aligned}\right\} \quad (A.14)$$

For the case of homogeneous and ergodic processes, the property of equivalence of space and probability averages generally applies and

$$\left.\begin{aligned} \text{Av.}\{f(x,y)\} &= \lim_{\substack{X\to\infty \\ Y\to\infty}} \frac{1}{4XY} \int_{-Y}^{Y} \int_{-X}^{X} f(x,y) dx\ dy \\ \text{Av.}\{f^2(x,y)\} &= \lim_{\substack{X\to\infty \\ Y\to\infty}} \frac{1}{4XY} \int_{-Y}^{Y} \int_{-X}^{X} f^2(x,y) dx\ dy \\ \text{Av.}\{f(x_1,y_1)\ f(x_2,y_2)\} &= \lim_{\substack{X\to\infty \\ Y\to\infty}} \frac{1}{4XY} \int_{-Y}^{Y} \int_{-X}^{X} f(x_1,y_1)\ f(x_2,y_2) dx_1\ dy_1 \\ &= R_{ff}(\Delta x, \Delta y) \\ \text{where } \Delta x = x_2 - x_1 &\text{ and } \Delta y = y_2 - y_1. \end{aligned}\right\} \quad (A.15)$$

Two-dimensional power spectra may also be defined by the reciprocal relations:

$$\left.\begin{aligned} \Phi_{ff}(\Omega_1,\Omega_2) &= \frac{1}{\pi^2} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[R_{ff}(\Delta x, \Delta y) e^{-i(\Omega_1 \Delta x + \Omega_2 \Delta y)} \right] d\Delta x\ \ d\Delta y \\ R_{ff}(\Delta x, \Delta y) &= \frac{1}{4} \int_{-\infty}^{\infty} \int_{-\infty}^{\infty} \left[\Phi_{ff}(\Omega_1,\Omega_2) e^{i(\Omega_1 \Delta x + \Omega_2 \Delta y)} \right] d\Omega_1\ \ d\Omega_2 \end{aligned}\right\} \quad (A.16)$$

3.2 Input-Output Relations

For an airplane flying along the x axis at a given speed through two-dimensional turbulence $f(x,y)$, the response along the flight path for a linear system may be expressed as

$$z(x) = \int_{-b/2}^{b/2}\int_{-\infty}^{\infty} f(x - x_1, y)\, h(x_1, y)dx_1\, dy \tag{A.17}$$

where b is the airplane span and $h(x,y)$ is the airplane response along the flight path to a unit impulse gust impinging on the wing leading edge at $x = 0$ and at span position y. Equation (A.17), as indicated in Reference 22, leads to the following relation between the power spectra of the disturbances and the response

$$\Phi_{zz}(\Omega_1) = \int_{-\infty}^{\infty} |H(\Omega_1,\Omega_2)|^2\Phi_{ff}(\Omega_1,\Omega_2)d\Omega_2 \tag{A.18}$$

where

$$H(\Omega_1,\Omega_2) = \int_{-b/2}^{b/2}\int_{-\infty}^{\infty}\left[h(x,y)e^{-i(\Omega_1 x+\Omega_2 y)}\right] dx\, dy$$

and defines the airplane response to unit sinusoidal gusts of frequency Ω_1 along the x axis and frequency Ω_2 along the y axis ($f(x,y) = (\sin \Omega_1 x)\ (\sin \Omega_2 y)$).

3.3 Isotropic Turbulence

For the special case of isotropic turbulence

$$R_{ff}(\Delta x, \Delta y) = \tilde{R}_{ff}\sqrt{(\Delta x^2 + \Delta y^2)} = R_{ff}(r) \tag{A.19}$$

and

$$\Phi_{ff}(\Omega_1,\Omega_2) = \tilde{\Phi}_{ff}(\Omega) \tag{A.20}$$

where

$$r^2 = \Delta x^2 + \Delta y^2$$

$$\Omega^2 = \Omega_1^2 + \Omega_2^2$$

Thus the condition of isotropy permits the important reduction of the two-dimensional auto-correlation functions and power spectra of Equations (A.14) and (A.16) of this Appendix to one-dimensional functions. Alternatively, the determination of the auto-correlation function $R_{ff}(\Delta x)$ from flight tests permits the determination of the two-dimensional spectrum $\tilde{\Phi}_{ff}(\Omega)$ by means of the following results given in Reference 22

$$\tilde{\Phi}(\Omega) = \frac{2}{\pi}\int_0^{\infty} \Delta x J_0(\Omega\Delta x)R_{ff}(\Delta x)d\Delta x \tag{A.21}$$

where $J_0(\ )$ is the Bessel function of the first kind of order 0. Thus, the single flight runs may be used to determine the spectral characteristics of the two-dimensional process $f(x,y)$

3.4 Turbulence Components

In studying the airplane response to atmospheric turbulence, it is frequently required to consider the effect of the several directional components of the turbulence. Thus, $f(x,y)$ must be considered a vector process with components $u(x,y)$, $v(x,y)$, and $w(x,y)$ along the x, y, and z axes, respectively. The statistical characteristics defined earlier for the scalar function have then to be defined for each of the three components as well as for six cross-component terms. These cross-component terms involve averages of the form:

$$\text{Av.}\{u(x_1,y_1)\ v(x_2,y_2)\} = R_{uv}(\Delta x, \Delta y)$$

The following auto-covariance tensor covering the nine functions which are required is of concern:

$$R(\Delta x, \Delta y) = \begin{vmatrix} R_{uu} & R_{uv} & R_{uw} \\ R_{vu} & R_{vv} & R_{vw} \\ R_{wu} & R_{wv} & R_{ww} \end{vmatrix}$$

For the case of isotropic turbulence, von Kármán and Howarth[28] have shown that these functions may all be expressed in terms of a single scalar function $f(r)$ which depends only on the radial distance $r^2 = \Delta x^2 + \Delta y^2$. The relations are

$$\left.\begin{aligned}
R_{uu} &= \frac{f(r) - g(r)}{r^2}(\Delta x)^2 + g(r) \\
R_{uv} &= R_{vu} = \frac{f(r) - g(r)}{r^2}(\Delta x)(\Delta y) \\
R_{vv} &= \frac{f(r) - g(r)}{r^2}\Delta y^2 + g(r) \\
R_{uw} &= R_{wu} = R_{vw} = R_{wv} = 0 \\
R_{ww} &= g(r)
\end{aligned}\right\} \qquad (A.22)$$

where $g(r)$ and $f(r)$ are the covariances between the velocity components normal to and parallel to the vector r. The functions $f(r)$ and $g(r)$ are also related by the following differential equation:

$$g(r) = f(r) + \frac{1}{2}r\frac{df(r)}{dr} \qquad (A.23)$$

Thus, for isotropic turbulence, the auto-correlation functions between the turbulence components at various different positions depends only upon the single function $f(r)$.

The relation between the auto-covariances given by Equation (A.23) may also be used to derive the relation between the power spectra of the velocity components. In terms of these power spectra, the relation is given by

$$G(\Omega) = \frac{1}{2} F(\Omega) - \frac{1}{2} \Omega \frac{dF(\Omega)}{d\Omega} \qquad (A.24)$$

where $G(\Omega)$ is the power spectrum of the lateral gust component and $F(\Omega)$ is the power spectrum of the longitudinal gust component. The vertical and side gust components sensed by an airplane are lateral components, while the gust component sensed by a sensitive pitot head is the longitudinal component. Equation (A.24) may thus be used to determine one of these spectra from measurements of the other.

Equations (A.23) and (A.24) are difficult to apply in checking on questions of isotropy for experimental measurements since the derivatives of $f(r)$ and $F(\Omega)$ are difficult to determine experimentally. However, idealized analytic approximations to the measured spectra may be used as indicated in the text to examine measurements of $F(\Omega)$ and $G(\Omega)$ in order to determine whether the isotropic condition imposed by Equation (A.24) is met over the frequency range covered by the data. In practice, it might be expected that isotropy will, in general, apply over a restricted frequency region.

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